International Journal of Thermophysics, Vol. 1, No. 2, 1980

Determination of Thermal Diffusivity of Solids by Use of Periodic Heat Flow

M. Lamvik[1]

Received November 13, 1979

This paper deals with the determination of thermal diffusivity in a cylindrical specimen by the use of a periodic heat flow in the axial direction. Heat transfer from the periphery is taken into account, and its influence upon the evaluation of thermal diffusivity from measurement of phase lag and amplitude decrement, respectively, is discussed. Experimental conditions are pointed out for which the evaluation can be done as for a semiinfinite specimen. Theoretical considerations are compared with experimental results.

KEY WORDS: periodic heat flow; thermal diffusivity; transport properties.

1. INTRODUCTION

The thermal diffusivity of a material relates its heat conductivity to its volumetric heat capacity. The diffusivity is an important transport property and is involved in many engineering problems of heat transfer. It is the determining parameter in the differential equation for a nonstationary temperature field in a solid. If the diffusivity is known, the temperature field may be evaluated from the solution of the differential equation and its boundary conditions.

On the other hand, if the temperature field is known by measurement, then the thermal diffusivity may be derived from the same equation. This has led to the development of several methods for the experimental determination

[1]Institutt for Teknisk Varmelaere, Universitetet i Trondheim, N.T.H., Trondheim, Norway.

0195-928X/80/0600-0233$03.00/0

NOMENCLATURE

$a = k/\rho c$	thermal diffusivity (m^2/s)
c	specific heat (J/kg °C)
F	heat loss factor (–)
h	heat transfer coefficient (W/m^2 °C)
k	thermal condictivity (W/m °C)
$P = 2\pi/\omega$	period of temperature variation (s)
r	radius of cross-section of specimen (m)
t	time (s)
Δt	time lag (s)
T	temperature (°C)
x	axial coordinate in specimen (m)
Δx	axial distance between measuring points (m)
$\mathrm{Bi} = (h \cdot r)/k$	Biot's number
$\mathrm{Fo} = (a \cdot P)/r^2$	Fourier's number
$\theta = T_t - T_s$	temperature difference (°C)
$\lambda = 1/\pi \cdot \mathrm{Bi} \cdot \mathrm{Fo}$	heat loss parameter (–)
μ	heat loss coefficient (s^{-1})
ρ	density (kg/m^3)
ω	frequency of the periodic temperature variation (s^{-1})
$\Delta\theta$	amplitude of temperature variation (°C)

Subscripts

a	amplitude decrement
1	phase lag
0	thermal upstream measuring point
s	stationary temperature
t	time-dependent temperature

of diffusivity. A comprehensive survey of the relevant literature may be found in [1].

The methods can be divided into two groups, according to the nature of the temperature field in the test specimen, namely the periodic temperature method and the transitory temperature method. The methods have been applied to test specimens of regular geometrical shape and under thermal conditions that allow the specimens to be treated mathematically as idealized bodies, such as long rods, flat plates, and semiinfinite solids or cylinders. In other words, the test specimens have to fit the method. In engineering practice, however, the specimen is more often unique and of given shape. An example is cylindrical rods obtained from core drilling in rocks or refractory materials. Further work on shaping the rods may not be convenient, and it may also affect the uniqueness of the specimen. In such cases, it is essential to have a method for the determination of the thermal diffusivity that is applicable to any specimen.

This paper treats the periodic temperature method as applied to test specimens of cylindrical form with periodic heat flow in the axial direction. The aim is to establish the experimental conditions under which the rod can be handled mathematically as a semiinfinite body. If the rod can be considered semiinfinite in the axial direction, any radial heat loss will not influence the calculation, and the thermal diffusivity can be evaluated from one set of measurements. When heat loss has to be taken into account, several sets of measurements are needed. Experimental runs must be performed using different values of the independent variables to make it possible either to eliminate the influence of heat loss by calculations as King did [2], or to find the conditions at which the thermal diffusion is revealed as uniform by further variation of the parameters. King needed two sets of measurements for his evaluation of the diffusivity. The alternate object needs more measurements.

The following shows how data for the experimental conditions can be treated to demonstrate the influence of heat loss, if any, and thereby also clarify the conditions where only one set of measurements is sufficient for evaluation of the thermal diffusivity.

2. THE DIFFERENTIAL EQUATION

The treatment here is mainly based on the analysis by Danielson and Sidles [5]. We consider a cylindrical rod that is heated at one end and cooled at the other. Heat flows from the heated to the cooled end. The heat source produces a stationary temperature field on which is superimposed a periodic field, which varies as the cosine of the time.

The temperature field in the rod for the one-dimensional case can be described mathematically by the equation

$$c\rho \cdot \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} + Q_s(x) + Q_p(t) \tag{1}$$

where $Q_s(x)$ is the stationary heat loss from the periphery to the surroundings, and $Q_p(t)$ is the time-dependent heat loss that is referred to the time-dependent temperature of the rod. These two terms for heat loss are related to the respective temperature fields. Equation (1) describes two fields, the stationary and the time-dependent, which are additive. By subtracting terms describing the stationary field from Eq. (1), we get the equation for the time-dependent temperature field. This equation can be written

$$\frac{\partial \theta}{\partial t} = a \cdot \frac{\partial^2 \theta}{\partial x^2} - \mu\theta \tag{2}$$

where $\theta = T_t - T_s$ is the time-dependent temperature rise above the stationary temperature T_s at a certain location, and μ is the heat loss coefficient related to the heat capacity of the rod. With the boundary conditions

$$\begin{aligned} \theta(0, t) &= \theta_0 \cos \omega t \\ \theta(\infty, t) &= 0 \end{aligned} \tag{3}$$

Equation (2) has a solution

$$\theta(x, t) = \theta_0 \cdot e^{-px} \cdot \cos(\omega t - qx) \tag{4}$$

where

$$p = \left(\frac{\mu + \sqrt{\mu^2 + \omega^2}}{2a} \right)^{1/2} \tag{5}$$

and

$$q = \left(\frac{-\mu + \sqrt{\mu^2 + \omega^2}}{2a} \right)^{1/2} \tag{6}$$

The solution (4) describes a temperature field that has the form of a wave propagating in the axial direction of the rod, and whose phase lag and amplitude are influenced by the heat loss to the surroundings.

From the expression for the temperature field (4), two expressions for the thermal diffusivity may be extracted as functions of measurable quantities, namely, the phase lag and the amplitude decrement of the temperature wave. These expressions are treated separately.

2.1. Criteria for the Use of Phase Lag

To study how a phase of the temperature wave propagates through the rod, the maximal value of a certain wave is chosen as the reference event; at distance x_1 from the heat source. The maximal value occurs when

$$\cos(\omega t_1 - q_1 x_1) = 0 \tag{7}$$

or

$$t_1 = \frac{q_1 x_1}{\omega} \tag{8}$$

At another distance $x_2 > x_1$, the same phase occurs at the time

$$t_2 = \frac{q_2 x_2}{\omega} \tag{9}$$

The time difference

$$\Delta t = t_2 - t_1 = \frac{1}{\omega}(q_2 x_2 - q_1 x_1) \tag{10}$$

is the phase lag on the temperature wave while it propagates from x_1 to x_2.

It is here assumed that the heat loss coefficient is independent of x between x_1 and x_2, that is, $q_1 = q_2$. Then the expression for the phase lag can be written as

$$\Delta t = \frac{\Delta x}{\sqrt{2a \cdot \omega}}\left(\sqrt{\left(\frac{\mu}{\omega}\right)^2 + 1} - \frac{\mu}{\omega}\right) \tag{11}$$

The expression gives the thermal diffusivity as

$$a = \frac{\Delta x^2}{2\omega \cdot \Delta t^2}\left(\sqrt{\left(\frac{\mu}{\omega}\right)^2 + 1} - \frac{\mu}{\omega}\right) = \frac{\Delta x^2}{2\omega \cdot \Delta t^2} F_1 \tag{12}$$

F_1 is introduced as a heat loss factor related to the phase lag in the temperature wave. F_1 is always ≤ 1. When heat losses are negligible, $\mu = 0$; then $F_1 = 1$, and the expression for a is identical to that for the semiinfinite test specimen. When heat losses are present, then $F_1 < 1$, and the measured value of the diffusivity is apparently too low. Heat is then diffusing radially out of the rod between the observation points x_1 and x_2.

For a further analysis of the heat loss factor, the following expressions are introduced:

$P = 2\pi/\omega$	the period of the temperature wave
$h = (\mu/2)\, r \cdot \rho c$	the radial heat transfer coefficient at the cylindrical boundary of the rod

$\mathrm{Bi} = (h \cdot r)/k$ the Biot number for the radial heat transfer
$\mathrm{Fo} = (a \cdot P)/r^2$ the Fourier number for the radial heat transfer

The expression for h is deduced from the fact that the heat loss from an element of the rod is equal to the heat flow from the periphery, that is,

$$\pi r^2 \cdot \Delta x \cdot \rho c \cdot \theta \cdot \mu = 2\pi r \cdot \Delta x \cdot h \cdot \theta \tag{13}$$

μ is here seen to be the rate at which stored heat is removed from the rod. These expressions are put into the quotient μ/ω to give:

$$\lambda = \frac{\mu}{\omega} = \frac{1}{\pi}\,\mathrm{Bi} \cdot \mathrm{Fo} \tag{14}$$

With this formula, the heat loss factor can be writen

$$F_1 = \sqrt{\lambda^2 + 1} - \lambda \tag{15}$$

When the heat transfer between the rod and the surroundings is low, $h \longrightarrow 0$; then $\lambda \ll 1$ and $F_1 \longrightarrow 1$. The termal diffusivity can in that case be evaluated from the expression for the semiinfinite specimen. From this we see that the product of the Biot number and the Fourier number, as they are defined here, governs the experimental conditions under which the rod can be treated as a semiinfinite body.

In Fig. 1, the factor F_1 is shown as function of λ. From the figure we can see, for example, that the thermal diffusivity can be evaluated by measurement to within 10% when $\lambda \leqq 10^{-1}$ and to within 1% when $\lambda \leqq 10^{-2}$.

2.2. Criteria for Use of Amplitude Decrement

From the expression (4) it can be seen that the imagined temperature wave propagates a certain axial distance Δx in the rod with an amplitude decrement of

$$\frac{\Delta\theta}{\Delta\theta_0} = \exp\left[-\left(\frac{\mu + \sqrt{\mu^2 + \omega^2}}{2a}\right)^{1/2} \cdot \Delta x\right] \tag{16}$$

This expression gives the thermal diffusivity explicitly as

$$a = \frac{\pi \Delta x^2}{P \cdot \ln^2(\Delta\theta/\Delta\theta_0)} \cdot F_a \tag{17}$$

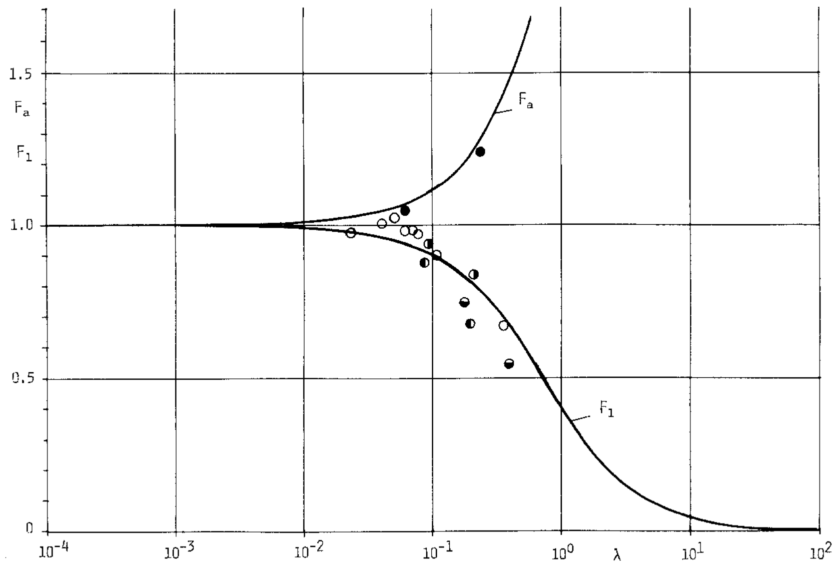

Fig. 1. Heat loss factors as functions of λ. ◐, F_1, from King's experiments (Cu, 35°C); ◑, F_1, from King's experiments (Cu, 60°C); ⊖, F_1, from King's experiments (Sn, 35°C); ○, F_1, from the author's experiments (Al, 60°C); ●, F_a, from the author's experiments (Al, 60°C).

where F_a is a heat loss factor related to the logarithmic amplitude decrement. After manipulation of expressions similar to that given in the previous section, the heat loss factor can be expressed as

$$F_a = \sqrt{\lambda^2 + 1} + \lambda \tag{18}$$

F_a is shown in Fig. 1 as a function of λ.

From the figure, we can see, for examle, that when $\lambda \leq 10^{-2}$, the thermal diffusivity can be evaluated to within 1% of the value given from equations for a semiinfinite specimen. The thermal diffusivity determined from experiments involving high values of λ will be too high when evaluated from the expression for the semiinfinite specimen.

2.3. General Criteria

The heat loss factors can be seen to be independent of the heat conductivity of the test specimen. These factors depend on the heat capacity together with the impressed variables, wave period P, heat transfer coefficient h, and radius of the rod's cross-section.

The factors are related as

$$\frac{F_1}{F_a} = \left(\frac{\omega \Delta t}{\ln (\Delta\theta/\Delta\theta_0)}\right)^2 \tag{19}$$

For a semiinfinite body, $F_1 = F_a = 1$, and we have from Eq. (19) the condition that

$$\frac{P}{\Delta t} \ln \frac{\Delta\theta}{\Delta\theta_0} = -2\pi \tag{20}$$

This relation will serve as a guide in ensuring that the measurements, phase lag and amplitude decrement, will allow for the thermal diffusivity to be evaluated as for a semiinfinite specimen. The condition (20) is fulfilled when the parameter $\lambda = (1/\pi)$ Bi $\cdot$ Fo $\leq 10^{-2}$, and gives values of the diffusivity within about 1%, due to heat losses.

3. PRACTICAL CONSIDERATIONS

Practical considerations are mainly concerned with the choice of distance Δx between the temperature measuring device and the period P. The choices are made with the intention to minimize the overall uncertainty in evaluated values of the diffusivity.

The temperatures are usually measured by use of thermocouples. The thermocouples may be placed in radial bores that reach just to the axis of the specimen. A practical minimum diameter of the thermocouple is about 0.5 mm for temperatures <1000°C. The axial distance between the thermocouples can then be measured within an estimated total uncertainty of ± 0.4 mm. This uncertainty in Δx will be reflected in the relative uncertainty of the calculated value of the thermal diffusivity; for example, 5% for $\Delta x = 15$ mm or about 2% for $\Delta x = 50$ mm.

The choice of distance Δx has, however, to be a compromise between the accuracy of Δx values and the smallest amplitude of the temperature wave that can be registered at the downstream measuring point. The amplitude at that point will for a given material in a given experiment be dependent on the period of the wave, a period that further has to be chosen so that the heat loss factors can be rendered approximately 1.0, according to Fig. 1. In the final choice, a reasonable distance should be related as

$$\Delta x / \sqrt{k \cdot r} = \text{constant} \tag{21}$$

This means that specimens of a material with low conductivity should have a larger cross-section than specimens of materials with high conductivity for a given period.

4. COMPARISON WITH EXPERIMENTAL RESULTS

The heat loss factors as they have been defined here can be evaluated from experimental data for specimens of known thermal diffusivity. The results of King's experiment [2] are available for comparison with the theoretical results. King determined the thermal diffusivities of tin and copper using specimens in the form of wires, 0.0025 m in diameter. He measured the phase lag of the temperature variation between two points on the wire applying alternative frequencies of the temperature wave. From these results, we can calculate the loss factor as the ratio of the thermal diffusivity, evaluated from the measurement, and the known value for the diffusivity.

The heat loss from the wires can also be estimated. Nusselt's number for the heat transfer to the surrounding air is assumed to be Nu = 1.7 for wire temperature 35°C, and Nu = 1.8 for 60°C [3]. It is thereby possible to estimate the λ values corresponding to different values of the loss factor. The results are shown as plots in Fig. 1. It can be seen that the heat loss factor decreases approximately as predicted by the theoretical curve.

In the figure are also plotted results from some measurements on pure aluminum, made by the author [4]. The test specimen was cylindrical, diameter 20 mm, length 150 mm. The surface was insulated with asbestos tape, wrapped to a thickness of 4 mm, giving an overall heat transfer coefficient to the surrounding tube wall of about 40 W/m^2 °C. The period was varied over the range $40 < P < 888$ s. The plots indicate, respectively, the decrease in F_1 and the increase in F_a when $\lambda > 10^{-2}$.

5. CONCLUDING REMARKS

The periodic temperature method for determining the thermal diffusivity of solids is suitable for cylindrical specimens of various shapes. The influence of heat loss from specimen to surroundings can be controlled by the choice of experimental variables. When the heat loss parameter λ has values $\leq 10^{-2}$, then the specimen can be recognized as semiinfinite. When $\lambda \geq 10^{-1}$, the heat loss factor must be evaluated. Then the estimation of the radial heat transfer coefficient, together with the specific heat capacity of the specimen, becomes quite important. The method also makes possible the determination of the heat transfer coefficient between the outer surface of a specimen and its surroundings if specimens with known thermal diffusivity are used.

REFERENCES

1. R. P. Tye, ed., *Thermal Conductivity,* Vol. 1 (Academic, New York, 1969).
2. R. W. King, *Phys. Rev,* **6**:437 (1915).
3. W. H. McAdams, *Heat Transmission,* 3rd ed. (McGraw-Hill, New York, 1954), p. 176.
4. M. Lamvik, Determination of thermal conductivity in aluminium-alloys, chamotte and electrode-materials for aluminium industry. Technical Report (in Norwegian) (1974).
5. G. C. Danielson and P. H. Sidles, in *Thermal Conductivity,* Vol. 2, R. P. Tye, ed. (Academic, New York, 1969), pp. 149–201.